FREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.com

CP PUBLICATION

SHORT TRICKS

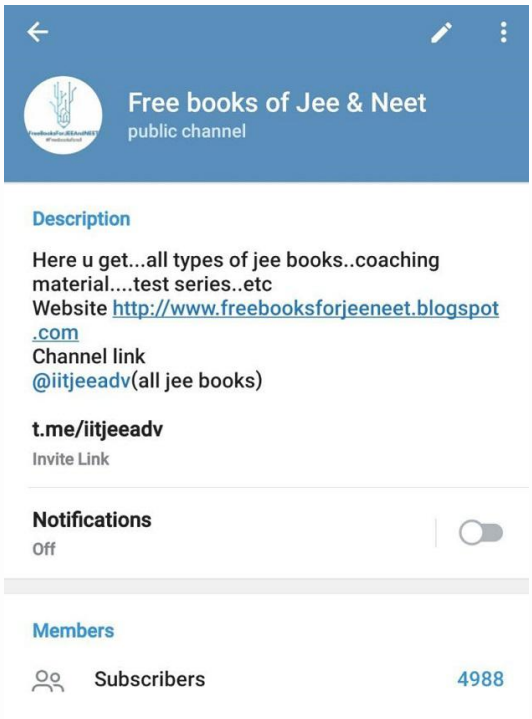
Mathematics

for JEE (Main & Adv)

Sunil Gupta
Career Point Editorial

FREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.com

SHORT TRICKS
Mathematics
for JEE (Main & Advanced)



CONTENTS

	Chapter	Page No.
1.	QUADRATIC EQUATION	01
2.	PROGRESSION	15
3.	COMPLEX NUMBER	35
4.	BINOMIAL THEOREM	51
5.	PERMUTATION & COMBINATION	68
6.	DETERMINANT	83
7.	TRIGONOMETRIC RATIO	110
8.	PROPERTIES OF TRIANGLE	134
9.	INVERSE TRIGONOMETRIC FUNCTION	146
10.	POINT & STRAIGHT LINE	159
11.	CIRCLE	184
12.	CONIC SECTIONS	197
13.	VECTOR	227
14.	FUNCTION	244
15.	LIMIT	280
16.	DIFFERENTIATION	288
17.	APPLICATION OF DERIVATIVE	295

17

Application of Derivative

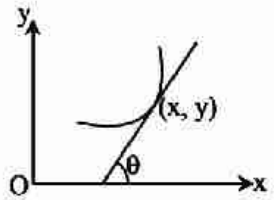
KEY CONCEPTS

Tangent & Normal

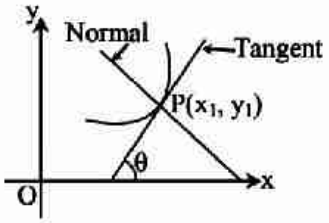
1. Geometrical Interpretation of the Derivative

Let $y = f(x)$ is a function then $\frac{dy}{dx}$ is slope of the tangent i.e. $\left(\frac{dy}{dx}\right)_{(x,y)} = \tan\theta$, where θ is the angle made by the tangent with the positive direction of x-axis.

or, $\theta = \tan^{-1}\left(\frac{dy}{dx}\right)$



2. Slope of the Tangent and Normal



Let $y = f(x)$ be a curve, and let $P(x_1, y_1)$ be a point on it. Then,

(i) slope of the tangent $= \left(\frac{dy}{dx}\right)_{(x_1,y_1)}$

(ii) Slope of the normal $= -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}} = -\left(\frac{dx}{dy}\right)_{(x_1,y_1)}$

(iii) The angle of inclination of tangent with x-axis $= \tan^{-1}\left(\frac{dy}{dx}\right)$

FREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.com

3. Equation of Tangent

Equation of tangent to the curve $y = f(x)$ at $P(x_1, y_1)$ is

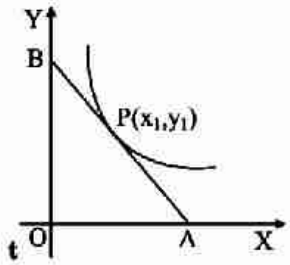
$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1)$$

(i) The tangent at (x_1, y_1) is parallel to x-axis $\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 0$

(ii) The tangent at (x_1, y_1) is parallel to y-axis $\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \infty$

(iii) The tangent line makes equal angles with the axes $\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \pm 1$

4. Length of Intercepts made on axes by the Tangent



$$\text{x-intercept} = OA = x_1 - \left\{\frac{y_1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}}\right\}$$

$$\text{y-intercept} = OB = y_1 - x_1\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$$

5. Length of Perpendicular from Origin to the Tangent

The length of perpendicular from origin (0, 0) to the tangent drawn at the point (x_1, y_1) of the curve $y = f(x)$ is

$$p = \left|\frac{y_1 - x_1\left(\frac{dy}{dx}\right)}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}\right|$$

6. Equation of Normal

The equation of normal at (x_1, y_1) to the curve $y = f(x)$ is

$$(y - y_1) = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}} (x - x_1)$$

(i) If normal is parallel to x-axis then $\Rightarrow \frac{dy}{dx} = \infty$

(ii) If normal is parallel to y-axis then $\Rightarrow \frac{dy}{dx} = 0$

(iii) If normal is equally inclined from both the axes or cuts equal intercept then $\Rightarrow \left(\frac{dy}{dx}\right) = \pm 1$

7. Length of Intercepts made on axes by the Normal

The length of intercept made by normal on x-axis is $x_1 + y_1\left(\frac{dy}{dx}\right)$

and length of intercept on y-axis is $y_1 + x_1\left(\frac{dx}{dy}\right)$

8. Length of Perpendicular from Origin to the Normal

The length of perpendicular from origin to normal is $p' = \left|\frac{x_1 + y_1\left(\frac{dy}{dx}\right)}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}\right|$

9. Equation of Tangent and Normal in "Parametric Form"

If $x = f(t)$ and $y = g(t)$ then equation of tangent is

$(y - g(t)) = \frac{g'(t)}{f'(t)}(x - f(t))$ and equation of normal is

$(y - g(t)) = -\frac{f'(t)}{g'(t)}(x - f(t))$

Type yFREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.comour text

10. Angle of Intersection of two Curves

If two curves $y = f_1(x)$ and $y = f_2(x)$ intersect at a point P, then the angle between their tangents at P is

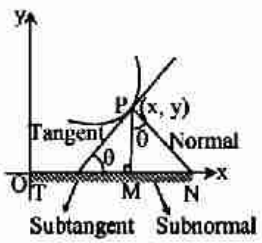
$\tan\phi = \pm\frac{(dy/dx)_1 - (dy/dx)_2}{1 + (dy/dx)_1(dy/dx)_2}$

The other angle of intersection will be $(180^\circ - \phi)$.

If two curves intersect orthogonally i.e. at right angle then they are called orthogonal curves and

$\left(\frac{dy}{dx}\right)_1 \cdot \left(\frac{dy}{dx}\right)_2 = -1$

11. Length of Tangent, Normal, Subtangent and Sub Normal



(i) Length of Tangent = PQ = $y \operatorname{cosec} \psi = \left| y \frac{\sqrt{1+(dy/dx)^2}}{(dy/dx)} \right|$

(ii) Length of the Normal = PR = $y \sec \psi = \left| y\sqrt{1+(dy/dx)^2} \right|$

(iii) Length of Sub tangent = QM $= y \cot \psi = \left| y/\left(\frac{dy}{dx}\right) \right|$

(iv) Length of Sub normal = MR = $y \tan \psi = \left| y\left(\frac{dy}{dx}\right) \right|$

12. Point of Inflexion

If at any point P, the curve is concave on one side and convex on other side with respect to x-axis, then the point P is called the point of inflexion. Thus P is a point of inflexion if at P,

$\frac{d^2y}{dx^2} = 0$, but $\frac{d^3y}{dx^3} \neq 0$

Also point P is a point of inflexion if $f''(x) = f'''(x) = ... = f^{n-1}(x) = 0$ and $f^n(x) \neq 0$ for odd n.

FREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.com

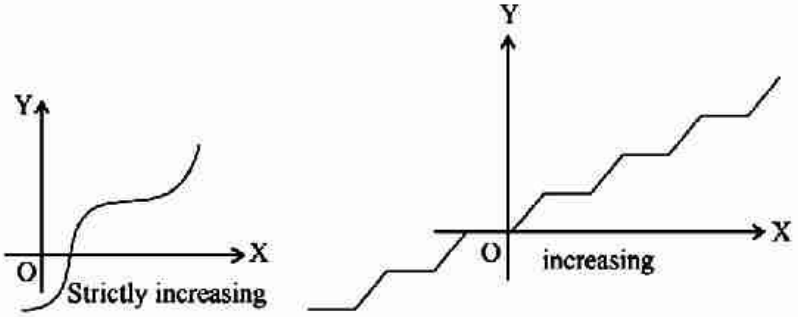
Monotonicity

1. Monotonic Function

These are of two types

(i) Monotonic Increasing

If the value of f(x) should increase (decrease) or remain equal by increasing (decreasing) the value of x.



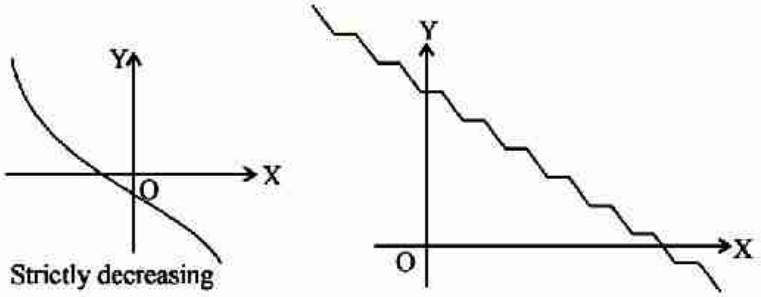
(we also say it non decreasing)

i.e. $\begin{cases} x_1 < x_2 \Rightarrow f(x_1) \le f(x_2) \\ \text{or } x_1 < x_2 \Rightarrow f(x_1) \ngeq f(x_2) \end{cases}, \ \forall x_1, x_2 \in D$

or $\begin{cases} x_1 > x_2 \Rightarrow f(x_1) \ge f(x_2) \\ \text{or } x_1 > x_2 \Rightarrow f(x_1) \nleq f(x_2) \end{cases}, \ \forall x_1, x_2 \in D$

(ii) Monotonic Decreasing

If the value of f(x) should decrease (increase) or remain equal by increasing (decreasing) the value of x.



Decreasing or monotonically decreasing (we say it non increasing)

i.e. $\begin{cases} x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2) \\ \text{or } x_1 < x_2 \Rightarrow f(x_1) \nleq f(x_2) \end{cases}, \ \forall x_1, x_2 \in D$

or $\begin{cases} x_1 > x_2 \Rightarrow f(x_1) \le f(x_2) \\ \text{or } x_1 > x_2 \Rightarrow f(x_1) \ngeq f(x_2) \end{cases}, \ \forall x_1, x_2 \in D$

Note :

- A function is said to be monotonic function in a domain if it is either monotonic increasing or monotonic decreasing in that domain.
- If $x_1 < x_2 \Rightarrow f(x_1) < f(x_2) \ \forall \ x_1, x_2 \in D$, then f(x) is called strictly increasing in domain D.
- If $x_1 < x_2 \Rightarrow f(x_1) > f(x_2), \ \forall \ x_1, x_2 \in D$ then it is called strictly decreasing in domain D.

2. Examples of Monotonic Function

Monotonic Increasing	Monotonic Decreasing	*Not Monotonic*
x^3	$1/x$	x^2
$x\|x\|$	$1-2x$	$\|x\|$
e^x	e^{-x}	$e^x + e^{-x}$
$\log_a x$, $a > 1$	$\log_a x$, $a < 1$	$\sin x$
$\tan x$	$\cot x$	$\cos x$
$\sinh x$	$\operatorname{cosech} x$	$\cosh x$
$[x]$	$\coth x$	$\operatorname{sech} x$

3. Method of Testing Monotonicity

(i) At a point $x = a$, function $f(x)$ is

Monotonic increasing $\Rightarrow f'(a) > 0$

Monotonic decreasing $\Rightarrow f'(a) < 0$

(ii) In an interval

A function $f(x)$ defined in the interval $[a, b]$ will be

Monotonic increasing	$\Rightarrow f'(x) \geq 0$	
Monotonic decreasing	$\Rightarrow f'(x) \leq 0$	
Constant	$\Rightarrow f'(x) = 0$	$\forall\, x \in (a, b)$
Strictly increasing	$\Rightarrow f'(x) > 0$	
Strictly decreasing	$\Rightarrow f'(x) < 0$	

Note

➢ A strictly monotomic function is always one one and onto or "BIJECTIVE".

4. Properties of monotonic function

(i) If $f(x)$ is strictly increasing function on an interval $[a, b]$, then f^{-1} exists and it is also a strictly increasing function.

(ii) If $f(x)$ is strictly increasing function on an interval $[a, b]$ such that it is continuous, then f^{-1} is continuous on $[f(a), f(b)]$.

(iii) If $f(x)$ is continuous on $[a, b]$ such that $f'(c) \leq 0$ ($f'(c) < 0$) for each $c \in (a, b)$, then $f(x)$ is monotonically (strictly) decreasing function on $[a, b]$

f	g	f(g(x))	g(f(x))
↑	↑	↑	↑
↓	↓	↑	↑
↑	↓	↓	↓
↓	↑	↓	↓

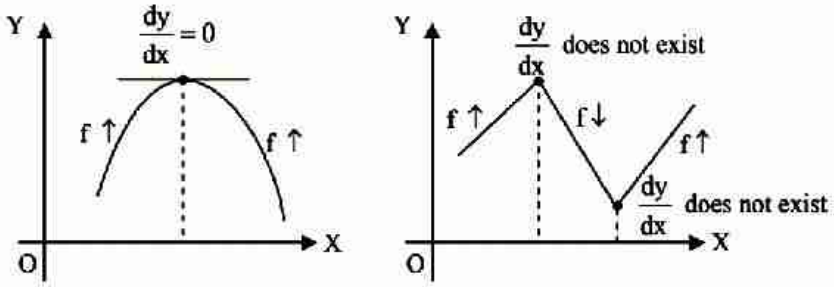
Where ↑ = Increasing ; ↓ = decreasing

FREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.com

Maxima & Minima

1. Necessary Condition to Exist an or more Extremum

For the existence of extremum either $f'(x) = 0$ or $f'(x)$ does not exist or both. **(Remember)**



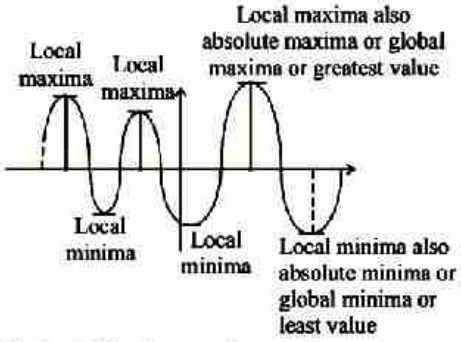
Note :

- The meaning of extremum is either a point of maximum or minimum.

2. Maximum & Minimum Points

(i) Maxima (Local Maxima) : A function f(x) is said to be maximum at x = a, if there exists a very small positive number h, such that
$f(x) < f(a) \forall x \in (a - h, a + h), x \neq a$

(ii) Minima (Local Minimum) : A function f(x) is said to be minimum at x = b, if there exists a very small positive number h, such that $f(x) > f(b)$, $\forall x \in (b - h, b + h), x \neq b$


Local maxima also absolute maxima or global maxima or greatest value
Local maxima
Local maxima
Local minima
Local minima
Local minima also absolute minima or global minima or least value

(iii) Absolute Maximum (Global Maximum) :
A function f has an absolute maximum (or global maximum) at c if $f(c) \geq f(x)$ for all x in D, where D is the domain of f. The number f(c) is called the maximum value of f on D.

(iv) Absolute minimum (Global Minimum) :
A function f has an absolute minimum at c if $f(c) \leq f(x)$ for all x in D and the number f(c) is called the minimum value of f on D. The maximum and minimum values of f are called the extreme values of f.

Note :

- The maximum and minimum points are also known as extreme points.
- A function may have more than one maximum and minimum points in an interval.
- A maximum value of a function f(x) in an interval [a, b] is not necessarily its greatest value in that interval. Similarly, a minimum value may not be the least value of the function.
- If a continuous function has only one maximum (minimum) point, then at this point function has its greatest (least) value.
- Monotonic functions do not have extreme points.

3. Conditions for Maxima & Minima of a Function

Necessary condition : A point x = a is an extreme point of a function f(x) if f'(a) = 0, provided f'(a) exists. Thus if f'(a) exists, then

x = a is an extreme point $\Rightarrow$ f'(a) = 0

or

f'(a) $\neq$ 0 $\Rightarrow$ x = a is not an extreme point.

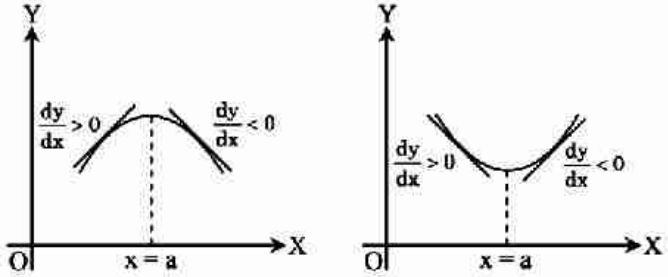
4. Derivative Test for Ascertaining Maxima/Minima

(i) First Derivative Test :

Find the point (say x = a) where f'(x) = 0 and

(a) If f'(x) changes sign from positive to negative while graph of the function passes through x = a then x = a is said to be a point of local maxima.

(b) If f'(x) changes sign from negative to positive while graph of the function passes through x = a then x = a is said to be a point of local minima.



(ii) Second order Derivative Test :

(ia) Find the differential coefficient of f(x) with respect to x, i.e. f'(x) and equate it to zero.

(bii) Find different real values of x by solving the equation f'(x) = 0. Let its roots be a, b, c, ...

(ciii) Find the value of f''(x) and substitute the value of a_1, a_2, a_3 ....in it and get the sign of f'(x) for each value of x.

(ivd) If f''(a) < 0 then the value of f(x) is maximum at x = a and if f''(a) > 0 then value of f(x) will be minimum at x = a. Similarly by getting the signs of f''(x) at other points b, c ..... we can find the points of maxima and minima.

(eiii) **n^{th} order Derivative Test**

If f'(a) = f''(a) = f'''(a) = ...= f^{n-1} (a) = 0 and f^n (a) $\neq$ 0 if n is odd then f(x) has neither local maximum nor local minimum at x = a and this is point of inflexion.

If n is even then f^n (a) < 0

f(x) has a local maximum at x = a and if f^n (a) > 0 then f(x) has a local minimum at x = a.

5. Greatest & Least Values of a Function in a given Interval

If a function f(x) is defined in an interval [a, b], then greatest or least values of this function occurs either at x = a or x = b or at those values of x where f'(x) = 0.

Thus greatest value of f(x) in interval [a, b] = max. [f (a), f(b), f(c)]

Least value of f(x) in interval [a, b] = Min. [f(a), f(b), f(c)]

where x = c is a point such that f'(c) = 0

6. Properties of Maxima & Minima

(i) The area of rectangle with given perimeter is greatest when it is a square.

(ii) The perimeter of a rectangle with given area is least when it is a square.

(iii) The greatest rectangle inscribed in a given circle is a square.

(iv) The greatest triangle inscribed in a given circle is equilateral.

(v) The semi vertical angle of a cone with given slant height and maximum volume is $\tan^{-1}\sqrt{2}$.

(vi) The height of a cylinder of maximum volume inscribed in a sphere of radius a is $2a/\sqrt{3}$.

(vii) For given perimeter equilateral triangle has greatest area.

(viii) For given area equilateral triangle has least perimeter.

(ix) $(a + b)^2 \le a^2 \sec^2\theta + b^2 \operatorname{cosec}^2\theta$

(x) A.M $\ge$ G.M.

7. Useful Formulae of Mensuration to Remember

(ai) Volume of a cuboid = ℓbh.

(bii) Surface area of a cuboid = $2(\ell b + bh + h\ell)$

(ciii) Volume of a prism = area of the base × height

(div) Lateral surface area of prism = perimeter of the base × height

(ev) Total surface area of a prism = lateral surface area + 2 area of the base
(Note that lateral surfaces of a prism are all rectangles.)

(fvi) Volume of a pyramid = $\frac{1}{3}$ area of the base × height.

(viig) Curved surface area of a pyramid = $\frac{1}{2}$(perimeter of the base) × slant height. (Note that slant surfaces of a pyramid are triangles)

(viiih) Volume of a cone = $\frac{1}{3}\pi r^2 h$

(ixi) Curved surface area of a cylinder = $2\pi rh$.

(xj) Total surface area of a cylinder = $2\pi rh + 2\pi r^2$

(xik) Volume of a sphere = $\frac{4}{3}\pi r^3$

(xiil) Surface area of a sphere = $4\pi r^2$

(xiiim) Area of a circular sector = $\frac{1}{2}r^2\theta$, when θ is in radians.

FREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.com

SHORT TRICKS

Trick -1 Equation of tangent to the curve $x^n + y^n = a^n$ at (x_1, y_1) is $\frac{x}{x_1^{(1-n)}} + \frac{y}{y_1^{(1-n)}} = a^n$

Q.1 The equation of tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point (x_1, y_1) is

(A) $\frac{x}{\sqrt{x_1}} + \frac{y}{\sqrt{y_1}} = \frac{1}{\sqrt{a}}$ (B) $x\sqrt{x_1} + y\sqrt{y_1} = \sqrt{a}$ (C) $\frac{x}{\sqrt{x_1}} + \frac{y}{\sqrt{y_1}} = \sqrt{a}$ (D) $\frac{x}{\sqrt{x_1}} + \frac{y}{\sqrt{y_1}} = a$

Sol. **[C]**

Proper Method

$\sqrt{x} + \sqrt{y} = \sqrt{a}$ (x_1, y_1) lies on curve

$\therefore \sqrt{x_1} + \sqrt{y_1} = \sqrt{a}$ .... (1)

Differentiate equation of curve. w.r.t. x

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{-\sqrt{y_1}}{\sqrt{x_1}}$$

Equation of tangent

$$\Rightarrow y - y_1 = \frac{dy}{dx}(x - x_1)$$

$$\Rightarrow y - y_1 = \frac{-\sqrt{y_1}}{\sqrt{x_1}}(x - x_1)$$

$$\Rightarrow y\sqrt{x_1} - y_1\sqrt{x_1} = -\sqrt{y_1}x + \sqrt{y_1}x_1$$

$$\Rightarrow x\sqrt{y_1} + y\sqrt{x_1} = \sqrt{y_1}x_1 + y_1\sqrt{x_1}$$

$\Rightarrow$ divide by $\sqrt{x_1}\sqrt{y_1}$

$$\Rightarrow \frac{x}{\sqrt{x_1}} + \frac{y}{\sqrt{y_1}} = \sqrt{x_1} + \sqrt{y_1}$$

$$\Rightarrow \frac{x}{\sqrt{x_1}} + \frac{y}{\sqrt{y_1}} = \sqrt{a}$$

Short Trick

Put $n = \frac{1}{2}$ in standard result

$$\frac{x}{\sqrt{x_1}} + \frac{y}{\sqrt{y_1}} = \sqrt{a}$$

Trick -2 If OA and OB are intercept made by tangent to the curve $x^m + y^m = a^m$ on the co-ordinate axes than $(OA)^n + (OB)^n = a^n$

Q.2 The sum of the intercepts made by a tangent to the curve $\sqrt{x} + \sqrt{y} = 4$ at point (4, 4) on coordinate axes is-

(A) $4\sqrt{2}$ (B) $6\sqrt{3}$ (C) $8\sqrt{2}$ (D) $\sqrt{256}$

Sol. **[D]**

Proper Method	Short Trick
$\sqrt{x} + \sqrt{y} = 4$ $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\sqrt{\frac{y}{x}}$ $\left(\frac{dy}{dx}\right)_{(4,4)} = -1$ $\therefore$ equation of tangent $y - 4 = -1\,(x - 4)$ $y - 4 = -x + 4 \Rightarrow x + y = 8$ $\Rightarrow \frac{x}{8} + \frac{y}{8} = 1$ $\therefore$ sum of intercepts on coordinate axis is $\therefore 8 + 8 = 16 = \sqrt{256}$	Curve $\sqrt{x} + \sqrt{y} = \sqrt{16}$ $OA + OB = 16 = \sqrt{256}$

Q.3 If the tangents at any point on the curve $x^4 + y^4 = a^4$ cuts off intercept p and q on the axes, the value of $p^{-4/3} + q^{-4/3}$ is-

(A) $a^{-4/3}$ (B) $a^{-1/2}$ (C) $a^{1/2}$ (D) None of these

Sol. **[A]**

Proper Method	Short Trick
$x^4 + y^4 = a^4$ .....(i) Let point $(x_1\ y_1)$ $\because x_1^4 + y_1^4 = a^4$ .....(ii) $\because$ equation of tangent at point (x_1y_1) on curve $x^n + y^n = a^n$ is $\frac{x}{x_1^{1-n}} + \frac{y}{y_1^{1-n}} = a^n$ $\therefore$ equation of tangent of (i) is $\frac{x}{x_1^{1-4}} + \frac{y}{y_1^{1-4}} = a^4$ $\Rightarrow \frac{x}{x_1^{-3} \cdot a^4} + \frac{y}{y_1^{-3} a^4} = 1$ $\therefore$ according to Q $p = x_1^{-3} a^4 \Rightarrow p^{-4/3} = x_1^4 a^{-16/3}$ $q = y_1^{-3} a^4 \Rightarrow q^{4/3} = y_1^4 a^{-16/3}$ $\therefore p^{-4/3} + q^{-4/3} = x_1^4 a^{-16/3} + y_1^4 (a^{-16/3})$ $= a^{-16/3}(x_1^4 + y_1^4)$ $= a^{-16/3} \cdot a^4$ $= a^{-16/3+4}$ $= a^{-4/3}$	Curve $x^4 + y^4 = a^4$ $p^{-4/3} + q^{-4/3} = a^{-4/3}$

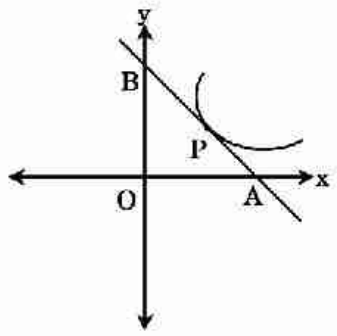
Trick -3 Area of triangle formed by tangent at any point of curve $2xy = a^2$ with axes is constant and it is equal to a^2

Q.4 Area of triangle formed by tangent at any point to the curve $xy = a^2$ with co-ordinate axes is -

(A) a^2 (B) $\frac{a^2}{2}$ (C) $2a^2$ (D) $\frac{a^2}{3}$

Sol. **[C]**

Proper Method



Curve $xy = a^2$

Let the point be (x_1, y_1)

$\therefore x_1y_1 = a^2$ .... (1)

Differentiation w.r.t. x

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{y_1}{x_1}$$

Equation of tangent

$$y - y_1 = \frac{-y_1}{x_1}(x - x_1)$$

intercept on x – axis : $y = 0 \Rightarrow x = 2x_1$

intercept on y – axis : $x = 0 \Rightarrow y = 2y_1$

Area of triangle $= \frac{1}{2}(2x_1)(2y_1)$

$= 2x_1y_1$

$= 2a^2$

Short Trick

If curve $2xy = a^2$ then area is a^2

So if curves $xy = a^2$

$\Rightarrow 2xy = 2a^2$

area is $2a^2$

Q.5 Area of triangle formed by tangent at any point to the curve $3xy = \frac{a^2}{2}$ is -

(A) a^2 (B) $\frac{a^2}{6}$ (C) $\frac{a^2}{3}$ (D) $\frac{a^2}{2}$

Sol. **[C]**

Proper Method

curve $3xy = \frac{a^2}{2}$

let the point be (x_1, y_1)

$$3x_1y_1 = \frac{a^2}{2} \quad \text{..... (1)}$$

Differentiate w.r.t x

Short Trick

If curve $2xy = a^2$ then area of triangle $= a^2$

so if curve $3xy = \frac{a^2}{2}$

$\Rightarrow 2xy = \frac{a^2}{3}$ then area of triangle $= \frac{a^2}{3}$

FREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.com

$3y + 3x\frac{dy}{dx} = 0$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{y_1}{x_1}$

Equation of tangent $\Rightarrow y - y_1 = \frac{-y_1}{x_1}(x - x_1)$

Intercept on x-axis : $y = 0 \Rightarrow x = 2x_1$

Intercept on y-axis : $x = 0 \Rightarrow y = 2y_1$

Area of triangle $= \frac{1}{2}(2x_1)(2y_1) = 2x_1y_1 = \frac{a^2}{3}$

Q.6 If the area of triangle included between the axes and any tangent to the curve $xy^n = a^{n+1}$ is constant then value of n is -

(A) -1 (B) 1 (C) 2 (D) -2

Sol. **[B]**

Proper Method

curve $xy^n = a^{n+1}$

let the point be (x_1, y_1)

$\therefore x_1 y_1^n = a^{n+1}$ .... (1)

Differentiate w.r.t. x

$y^n + xny^{n-1}\frac{dy}{dx} = 0$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{y_1}{nx_1}$

Equation of tangent $\Rightarrow y - y_1 = -\frac{y_1}{nx_1}(x - x_1)$

Intercept on x-axis : $y = 0 \Rightarrow x = x_1(1 + n)$

Intercept on y-axis : $x = 0 \Rightarrow y = y_1\frac{(1+n)}{n}$

Area of triangle $= \frac{1}{2}x_1(1 + n)y_1\frac{(1+n)}{n}$

from (1)

Area of triangle $= \frac{1}{2}\frac{a^{n+1}}{y_1^n}y_1\frac{(1+n)^2}{n}$

$= \frac{1}{2}\frac{a^{n+1}}{y_1^{n-1}}\frac{(1+n)^2}{n}$

For constant area $\Rightarrow n - 1 = 0$

$\Rightarrow n = 1$

Short Trick

If curve is $2xy = a^2$ then area $= a^2$ (constant)

If curve is $xy^n = a^{n+1}$

or $2xy^n = a^{n+1}$

and $n = 1 \Rightarrow 2xy = a^2$, then area $= a^2$ (constant)

Short Tricks

Trick -4 If two curves $a_1x^2 + b_1y^2 = 1$ and $a_2x^2 + b_2y^2 = 1$ cuts orthogonally to each other then

$$\frac{1}{a_1} - \frac{1}{a_2} = \frac{1}{b_1} - \frac{1}{b_2}$$

Q.7 If curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{\ell^2} - \frac{y^2}{m^2} = 1$ intersect orthogonally, then-

(A) $a^2 + b^2 = \ell^2 + m^2$ (B) $a^2 - b^2 = \ell^2 - m^2$

(C) $a^2 - b^2 = \ell^2 + m^2$ (D) $a^2 + b^2 = \ell^2 - m^2$

Sol. **[C]**

Proper Method

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ & $\frac{x^2}{\ell^2} - \frac{y^2}{m^2} = 1$

Let point of intersection is (x_1, y_1)

$\therefore \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$ ....(i)

& $\frac{x_1^2}{\ell^2} - \frac{y_1^2}{m^2} = 1$ ....(ii)

Now $\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$ & $\frac{2x}{\ell^2} - \frac{2y}{m^2}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{b^2}{a^2}\frac{x}{y}$ $\frac{dy}{dx} = \frac{m^2}{\ell^2}\frac{x}{y}$

$m_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{b^2}{a^2}\frac{x_1}{y_1}$ $m_2 = \left(\frac{dy}{dx}\right)_{(x_1y_1)} = \frac{m^2x_1}{\ell^2 y_1}$

$\because$ Angle of intersection is $\pi/2$

$\therefore m_1m_2 = -1$

$\frac{-b^2}{a^2}\frac{x_1}{y_1} \times \frac{m^2}{\ell^2}\frac{x_1}{y_1} = -1$

$\Rightarrow \frac{b^2}{a^2}\frac{m^2}{\ell^2}\frac{x_1^2}{y_1^2} = 1$

$\Rightarrow \frac{b^2m^2}{a^2\ell^2} = \frac{y_1^2}{x_1^2}$ ... (iii)

(i) – (ii)

$x_1^2\left(\frac{1}{a^2} - \frac{1}{\ell^2}\right) + y_1^2\left(\frac{1}{b^2} + \frac{1}{m^2}\right) = 0$

$\Rightarrow \frac{y_1^2}{x_1^2} = -\left(\frac{\ell^2 - a^2}{a^2\ell^2} \times \frac{b^2m^2}{b^2 + m^2}\right)$ .....(iv)

$\therefore$ (iii) & (iv) are same

$\therefore -\left(\frac{\ell^2 - a^2}{a^2\ell^2} \times \frac{b^2m^2}{b^2 + m^2}\right) = \frac{b^2m^2}{a^2\ell^2}$

$\Rightarrow -(\ell^2 - a^2) = b^2 + m^2$

$\Rightarrow a^2 - b^2 = \ell^2 + m^2$

Short Trick

$\frac{1}{1/a^2} - \frac{1}{1/\ell^2} = \frac{1}{1/b^2} - \frac{1}{-1/m^2}$

$\Rightarrow a^2 - \ell^2 = b^2 + m^2$

$\Rightarrow a^2 - b^2 = \ell^2 + m^2$

Trick -5 let $f(x) = \dfrac{a\sin x + b\cos x}{c\sin x + d\cos x}$

(i) If f(x) is increasing then $\begin{vmatrix} a & b \\ c & d \end{vmatrix} > 0$

(ii) If f(x) is decreasing then $\begin{vmatrix} a & b \\ c & d \end{vmatrix} < 0$

Q.8 Function $f(x) = \dfrac{\lambda\sin x + 3\cos x}{2\sin x + 6\cos x}$ is increasing when -

(A) $\lambda < 1$ (B) $\lambda > 1$ (C) $\lambda < 2$ (D) $\lambda > 2$

Sol. **[B]**

Proper Method

$$f(x) = \frac{\lambda\sin x + 3\cos x}{2\sin x + 6\cos x}$$

$$f'(x) = \frac{[2\sin x + 6\cos x][\lambda\cos x - 3\sin x] - [\lambda\sin x + 3\cos x](2\cos x - 6\sin x)}{(2\sin x + 6\cos x)^2}$$

$$= \frac{6\lambda - 6}{(2\sin x + 6\cos x)^2}$$

$\therefore$ f(x) is increasing

$$\therefore f'(x) > 0 \Rightarrow \frac{6\lambda - 6}{(2\sin x + 6\cos x)^2} > 0$$

$\Rightarrow 6\lambda - 6 > 0 \Rightarrow \lambda > 1$

Short Trick

$\begin{vmatrix} \lambda & 3 \\ 2 & 6 \end{vmatrix} > 0$

$\Rightarrow \lambda > 1$

Q.9 If $f(x) = \dfrac{\sin x + b\cos x}{\sin x + 4\cos x}$ is monotonic increasing them -

(A) $b < 8$ (B) $b < 4$ (C) $b > 4$ (D) $b > 8$

Sol. **[B]**

Proper Method

$$f(x) = \frac{\sin x + b\cos x}{\sin x + 4\cos x}$$

$$f'(x) = \frac{(\cos x - b\sin x)(\sin x + 4\cos x) - (\sin x + b\cos x)(\cos x - 4\sin x)}{(\sin x + 4\cos x)^2}$$

$$f'(x) = \frac{4 - b}{(\sin x + 4\cos x)^2}$$

Now f(x) will be monotonic increasing

when $f'(x) > 0$

$\Rightarrow 4 - b > 0$

$\Rightarrow b < 4$

Short Trick

Increasing function $\therefore \begin{vmatrix} 1 & b \\ 1 & 4 \end{vmatrix} > 0$

$\Rightarrow 4 - b > 0$

$\Rightarrow b < 4$

Trick -6 (i) If $f(x) = a\sin x \pm b\cos x$ → max. value $= \sqrt{a^2+b^2}$, min. value $= -\sqrt{a^2+b^2}$

(ii) If $f(x) = a^2\sec^2 x + b^2\operatorname{cosec}^2 x$ ⟶ min. $= (a+b)^2$

(iii) If $f(x) = a\sec x + b\operatorname{cosec} x$ ⟶ min. value $= (a^{2/3}+b^{2/3})^{3/2}$

Q.10 For the curve $\frac{c^4}{r^2} = \frac{a^2}{\sin^2\theta} + \frac{b^2}{\cos^2\theta}$, the maximum value of r is -

(A) $\frac{c^2}{a+b}$ (B) $\frac{a+b}{c^2}$ (C) $\frac{c^2}{a-b}$ (D) $c^2(a+b)$

Sol. **[A]**

Proper Method

$\frac{c^4}{r^2} = \frac{a^2}{\sin^2\theta} + \frac{b^2}{\cos^2\theta}$

$\frac{c^4}{r^2} = a^2\operatorname{cosec}^2\theta + b^2\sec^2\theta$

$r^2 = \frac{c^4}{a^2\operatorname{cosec}^2\theta + b^2\sec^2\theta}$

now, r^2 will be maximum when $a^2\operatorname{cosec}^2\theta + b^2\sec^2\theta$ will be minimum

let $y = a^2\operatorname{cosec}^2\theta + b^2\sec^2\theta$

$\frac{dy}{d\theta} = 2a^2\operatorname{cosec}\theta(-\operatorname{cosec}\theta\cot\theta) + 2b^2\sec\theta(\sec\theta\tan\theta)$

$\frac{dy}{d\theta} = -2a^2\frac{\cos\theta}{\sin^3\theta} + 2b^2\frac{\sin\theta}{\cos^3\theta}$

For maxima & minima

$\frac{dy}{d\theta} = 0 \Rightarrow \frac{-2a^2\cos\theta}{\sin^3\theta} + \frac{2b^2\sin\theta}{\cos^3\theta} = 0$

$\Rightarrow -2a^2\cos^4\theta + 2b^2\sin^4\theta = 0$

$\Rightarrow \tan^4\theta = \frac{a^2}{b^2} \Rightarrow \tan^2\theta = \frac{a}{b}$

Now $\frac{d^2y}{d\theta^2} > 0$ at this value of θ

So, $y_{min} = a^2\left(1+\frac{b}{a}\right) + b^2\left(1+\frac{a}{b}\right) = (a+b)^2$

$= \frac{c^4}{(a+b)^2}$

$r_{max} = \frac{c^2}{a+b}$

Short Trick

$\frac{c^4}{r^2} = \frac{a^2}{\sin^2\theta} + \frac{b^2}{\cos^2\theta}$

$\frac{c^4}{r^2} = a^2\operatorname{cosec}^2\theta + b^2\sec^2\theta$

$r^2 = \frac{c^4}{a^2\operatorname{cosec}^2\theta + b^2\sec^2\theta}$

$r^2_{max} = \frac{c^4}{(a^2\operatorname{cosec}^2\theta + b^2\sec^2\theta)\min}$

$= \frac{c^4}{(a+b)^2}$

$r_{max} = \frac{c^2}{a+b}$

Q.11 The greatest value of the function $y = \frac{\sin 2x}{\sin(x+\pi/4)}$; $x \in (0, \pi/2)$ is -

(A) 1 (B) $-\sqrt{2}$ (C) $\sqrt{2}$ (D) $1/\sqrt{2}$

FREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.com

Sol. [A]

Proper Method

$$z=\frac{\sin 2x}{\sin\left(x+\frac{\pi}{4}\right)};\ x\in\left(0,\frac{\pi}{2}\right)$$

$$=\frac{2\sin x\cos x}{\frac{\sin x}{\sqrt{2}}+\frac{\cos x}{\sqrt{2}}}=\frac{2\sqrt{2}}{\sec x+\operatorname{cosec} x}$$

Let $y = \sec x + \operatorname{cosec} x$

$$\frac{dy}{dx}=\sec x\tan x-\operatorname{cosec} x.\cot x$$

$$\frac{dy}{dx}=\frac{\sin x}{\cos^2 x}-\frac{\cos x}{\sin^2 x}$$

For maxima & minima $\frac{dy}{dx}=0$

$$\Rightarrow \frac{\sin x}{\cos^2 x}-\frac{\cos x}{\sin^2 x}=0$$

$\tan x = 1$

$$x=\frac{\pi}{4}$$

$$\text{at } x=\frac{\pi}{4}\Rightarrow\frac{d^2y}{dx^2}>0$$

$$\therefore z_{min}=\sqrt{2}+\sqrt{2}=2\sqrt{2}$$

$$y_{max}=\frac{2\sqrt{2}}{(\sec x+\operatorname{cosec} x)_{min}}=\frac{2\sqrt{2}}{(1^{2/3}+1^{2/3})^{3/2}}$$

$$=\frac{2\sqrt{2}}{2\sqrt{2}}=1$$

Short Trick

$$y=\frac{\sin 2x}{\sin\left(x+\frac{\pi}{4}\right)};\ x\in\left(0,\frac{\pi}{2}\right)$$

$$=\frac{2\sin x\cos x}{\frac{\sin x}{\sqrt{2}}+\frac{\cos x}{\sqrt{2}}}=\frac{2\sqrt{2}}{\sec x+\operatorname{cosec} x}$$

$$y_{max}=\frac{2\sqrt{2}}{(\sec x+\operatorname{cosec} x)_{min}}=\frac{2\sqrt{2}}{(1^{2/3}+1^{2/3})^{3/2}}$$

$$=\frac{2\sqrt{2}}{2\sqrt{2}}=1$$

Trick -7 If sum of two positive numbers is given and their product is maximum then divide the number into equal parts

Q.12 If sum of two positive numbers is 6 then maximum value of their product is -

(A) 3 (B) 6 (C) 9 (D) 12

Sol. [C]

Proper Method

Let the numbers are x and y given that $x + y = 6$

$\Rightarrow y = 6 - x$

Let $z = xy$

$z = x(6 - x) = 6x - x^2$

$$\frac{dz}{dx}=6-2x$$

for maxima $\frac{dz}{dx}=0\Rightarrow 6-2x=0$

$\Rightarrow x = 3$

$$\frac{d^2z}{dx^2}=-2<0$$

$\therefore$ max at $x = 3$

and maximum value of $z = 3(6 - 3) = 9$

Short Trick

$x + y = 6$

$x = y = 3$

$Z_{max} = (3)(3) = 9$

Q.13 If $A > 0, B > 0$ and $A + B = \frac{\pi}{6}$, then the minimum value of $\tan A + \tan B$ is : **JEE Main Online - 2016]**

(A) $\frac{2}{\sqrt{3}}$ (B) $4 - 2\sqrt{3}$ (C) $\sqrt{3} - \sqrt{2}$ (D) $2 - \sqrt{3}$

Sol. **[B]**

Proper Method

If $A > 0, B > 0$ and $A + B = \frac{\pi}{6}$ then

Let $z = \tan A + \tan B$

$z = \tan A + \tan\left(\frac{\pi}{6} - A\right)$

$\frac{dz}{dA} = \sec^2 A - \sec^2\left(\frac{\pi}{6} - A\right)$

for pt. of min.

$\Rightarrow \sec^2 A - \sec^2\left(\frac{\pi}{6} - A\right) = 0$

$\Rightarrow \sec^2 A = \sec^2\left(\frac{\pi}{6} - A\right)$

$\Rightarrow \cos^2 A = \cos^2\left(\frac{\pi}{6} - A\right)$

$\Rightarrow \cos^2 A - \cos^2\left(\frac{\pi}{6} - A\right) = 0$

$\Rightarrow \sin\left(\frac{\pi}{6} - A + A\right)\sin\left(\frac{\pi}{6} - A - A\right) = 0$

$\Rightarrow \sin\left(\frac{\pi}{6} - 2A\right) = 0$

$\Rightarrow A = \frac{\pi}{12}$ and $B = \frac{\pi}{12}$

at $A = \frac{\pi}{12}$ & $B = \frac{\pi}{12}$

$\frac{d^2z}{dA^2}$ is positive

So, $z_{min} = \tan\frac{\pi}{12} + \tan\frac{\pi}{12}$

$= 2\tan\frac{\pi}{12}$

$= 2 \times \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{2(\sqrt{3}-1)^2}{2}$

$= 3 + 1 - 2\sqrt{3}$

$= 4 - 2\sqrt{3}$

Short Trick

$A + B = \frac{\pi}{6}$

$A = B = \frac{\pi}{12}$

$(\tan A + \tan B)_{max} = \tan\frac{\pi}{12} + \tan\frac{\pi}{12}$

$= 2\tan\left(\frac{\pi}{12}\right)$

$= 2\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) = 4 - 2\sqrt{3}$

FREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.com

Trick -8 Finding maximum and minimum value by using A.M. ≥ G.M.

Q.14 If $xy = c^2$, then the minimum value of $ax + by$ $(a > 0, b > 0)$ is-

(A) $c\sqrt{ab}$ (B) $-c\sqrt{ab}$ (C) $2c\sqrt{ab}$ (D) $-2c\sqrt{ab}$

Sol. **[C]**

Proper Method

Given $xy = c^2$

$y = \frac{c^2}{x}$

let $z = ax + by$

$z = ax + b\left(\frac{c^2}{x}\right)$ ..... (1)

$\Rightarrow \frac{dz}{dx} = a - \frac{bc^2}{x^2}$

for maxima & minima

$\frac{dz}{dx} = 0 \Rightarrow a - \frac{bc^2}{x^2} = 0$

$\Rightarrow x = \pm c\sqrt{\frac{b}{a}}$

Now $\frac{d^2z}{dx^2} = \frac{2bc^2}{x^3}$

at $x = c\sqrt{\frac{b}{a}} \Rightarrow \frac{d^2z}{dx^2} > 0$

so minimum value of

$z = a\ c\sqrt{\frac{b}{a}} + bc^2\sqrt{\frac{a}{b}}.\frac{1}{c}$

$z = 2c\sqrt{ab}$

Short Trick

A.M ≥ G.M

$\Rightarrow \frac{ax + by}{2} \geq \sqrt{ax.by}$

$\Rightarrow ax + by \geq 2c\sqrt{ab}$ $(\because xy = c^2)$

Q.15 If $a^2x^4 + b^2y^4 = c^6$, then the maximum value of xy is -

(A) $\frac{c^3}{2ab}$ (B) $\frac{c^3}{\sqrt{2ab}}$ (C) $\frac{c^3}{ab}$ (D) $\frac{c^3}{\sqrt{ab}}$

Sol. **[B]**

Proper Method

Given $a^2x^4 + b^2y^4 = c^6$

$\Rightarrow y^4 = \frac{c^6 - a^2x^4}{b^2}$

Let $z = xy$

$w = z^4 = x^4y^4$

$\Rightarrow w = x^4\left(\frac{c^6 - a^2x^4}{b^2}\right)$ ..... (1)

$\Rightarrow w = \frac{x^4c^6 - a^2x^8}{b^2}$

$\frac{dw}{dx} = \frac{4x^3c^6 - 8x^7a^2}{b^2}$

Short Trick

A.M ≥ G.M

$\Rightarrow \frac{a^2x^4 + b^2y^4}{2} \geq \sqrt{a^2x^4b^2y^4}$

$\Rightarrow \frac{c^6}{2} \geq abx^2y^2$

$\Rightarrow \frac{c^3}{\sqrt{2ab}} \geq xy$

For maxima & minima

$$\frac{dw}{dx} = 0 \Rightarrow \frac{4x^3}{b^2}(c^6 - 2a^2x^4) = 0$$

$$\Rightarrow x^2 = \pm\sqrt{\frac{c^6}{2a^2}}$$

$$\frac{d^2w}{dx^2} < 0 \text{ at } x^2 = \sqrt{\frac{c^6}{2a^2}}$$

$$\therefore z_{max}^4 = \frac{c^6}{2a^2}\left\{\frac{c^6 - a^2\frac{c^6}{2a^2}}{b^2}\right\} \{\text{from (1)}\}$$

$$z_{max}^4 = \frac{c^{12}}{4a^2b^2}$$

$$z_{max} = \frac{c^3}{\sqrt{2ab}}$$

Q.16 If $xy = a^2$ and $S = b^2x + c^2y$ where a, b and c are constants then the minimum value of S is -

(A) abc (B) $bc\sqrt{a}$ (C) 2abc (D) none of these.

Sol. **[C]**

Proper Method

Given $xy = a^2$

$$\Rightarrow y = \frac{a^2}{x}$$

$$\Rightarrow S = b^2x + c^2y$$

$$\Rightarrow S = b^2x + c^2\frac{(a^2)}{x} \quad \ldots\ldots (1)$$

$$\Rightarrow \frac{dS}{dx} = b^2 - \frac{c^2a^2}{x^2}$$

For maxima & minima $\frac{dS}{dx} = 0$

$$\Rightarrow b^2 - \frac{c^2a^2}{x^2} = 0$$

$$\Rightarrow x = \pm\frac{ac}{b}$$

$$\Rightarrow \frac{d^2S}{dx^2} = \frac{2c^2a^2}{x^3}$$

$$\text{at } x = \frac{ac}{b} \Rightarrow \frac{d^2S}{dx^2} > 0$$

$\therefore$ from (1)

$$S_{max} = b^2\left(\frac{ac}{b}\right) + c^2a^2\left(\frac{b}{ac}\right)$$

$= abc + abc$

$= 2abc$

Short Trick

A.M $\geq$ G.M

$$\frac{b^2x + c^2y}{2} \geq \sqrt{b^2x.c^2y}$$

$\Rightarrow S \geq 2abc$ ($\because xy = a^2$)

Q.17 If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is- **[AIEEE 2007]**

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

Sol. **[D]**

Proper Method

Given $p^2 + q^2 = 1$

$\Rightarrow q = \sqrt{1-p^2}$

Let $z = p + q$

$z = p + \sqrt{1-p^2}$

$\frac{dz}{dp} = 1 + \frac{1}{2\sqrt{1-p^2}}(-2p) = 1 - \frac{p}{\sqrt{1-p^2}}$

$\Rightarrow \frac{dz}{dp} = \frac{\sqrt{1-p^2} - p}{\sqrt{1-p^2}}$

For maxima & minima

$\frac{dz}{dp} = 0 \Rightarrow \frac{\sqrt{1-p^2} - p}{\sqrt{1-p^2}} = 0$

$\Rightarrow \sqrt{1-p^2} = p$

$\Rightarrow 1 - p^2 = p^2$

$\Rightarrow p = \pm \frac{1}{\sqrt{2}}$

$\frac{d^2z}{dp^2} < 0$ at $p = \frac{1}{\sqrt{2}}$

$\therefore$ max value of $z = \frac{1}{\sqrt{2}} + \sqrt{1-\frac{1}{2}} = \sqrt{2}$

Short Trick

Let $p = \sin\theta$ and $q = \cos\theta$

$p + q = \sin\theta + \cos\theta$

$(p + q)_{max} = \sqrt{2}$

Q.18 Find the minimum and maximum value of $f(x, y) = 7x^2 + 4xy + 3y^2$ subjected to $x^2 + y^2 = 1$.

(A) $5, 5 - 2\sqrt{2}$ (B) $5 + 2\sqrt{2}, 5 - \sqrt{2}$

(C) $5 - 2\sqrt{2}, 5 + 2\sqrt{2}$ (D) None

Sol. **[C]**

Proper Method

$f(x, y) = 7x^2 + 4xy + 3y^2$

and $x^2 + y^2 = 1$

$\Rightarrow y = \sqrt{1-x^2}$

$f(x) = 7x^2 + 4x\sqrt{1-x^2} + 3(1 - x^2)$

$= 4x^2 + 4x\sqrt{1-x^2} + 3$

$f'(x) = 8x + 4\sqrt{1-x^2} - \frac{4x^2}{\sqrt{1-x^2}} = 0$

$\Rightarrow 2x\sqrt{1-x^2} = 2x^2 - 1$

$\Rightarrow 4x^2(1 - x^2) = 4x^4 - 4x^2 + 1$

$\Rightarrow 8x^4 - 8x^2 + 1 = 0$

Short Trick

Let $x = \sin\theta$, $y = \cos\theta$

$7x^2 + 4xy + 3y^2$

$= 7\sin^2\theta + 4\sin\theta.\cos\theta + 3\cos^2\theta$

$= 7\left(\frac{1-\cos 2\theta}{2}\right) + 2\sin 2\theta + 3\left(\frac{1+\cos 2\theta}{2}\right)$

$= 2\cos 2\theta + 2\sin 2\theta + 5$

Max value $= 5 + 2\sqrt{2}$, min value $= 5 - 2\sqrt{2}$

$x^2 = \frac{8 \pm \sqrt{64-32}}{16} = \frac{8 \pm 4\sqrt{2}}{16}$

$x^2 = \frac{2 \pm \sqrt{2}}{4}$

$\because x^2 = \frac{2+\sqrt{2}}{4}, y^2 = \frac{2-\sqrt{2}}{4}$

and $x^2 = \frac{2-\sqrt{2}}{4}, y^2 = \frac{2+\sqrt{2}}{4}$

Solving we get maximum $= 5 - 2\sqrt{2}$

and minimum $= 5 + 2\sqrt{2}$

Q.19 If $\frac{x}{a} + \frac{y}{b} = 1$, then minimum value of $x^2 + y^2$ is-

(A) $\frac{2a^2b^2}{a^2+b^2}$ (B) $\frac{a^2b^2}{a^2+b^2}$ (C) $\frac{2ab}{a^2+b^2}$ (D) None of these

Sol. **[B]**

Proper Method

$\frac{x}{a} + \frac{y}{b} = 1$ ...(1)

Let $z = x^2 + y^2 = x^2 + \left(b\left(1 - \frac{x}{a}\right)\right)^2$ from (1)

$\frac{dz}{dx} = 2x + b^2 \cdot 2\left(1 - \frac{x}{a}\right)\left(-\frac{1}{a}\right)$

For maxima & minima $\frac{dz}{dx} = 0$

$\Rightarrow 2\left[x - \frac{b^2}{a} + \frac{b^2 x}{a^2}\right] = 0$

$\Rightarrow x\left(1 + \frac{b^2}{a^2}\right) = \frac{b^2}{a}$

$\Rightarrow x\left(\frac{a^2+b^2}{a^2}\right) = \frac{b^2}{a}$

$x = \frac{ab^2}{a^2+b^2}$

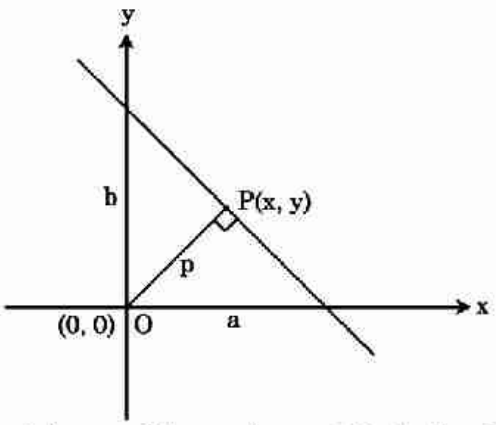
$\frac{d^2z}{dx^2} > 0$ at $x = \frac{ab^2}{a^2+b^2}$

$\therefore$ z will be minima at $x = \frac{ab^2}{a^2+b^2}$

$z_{min} = \left(\frac{ab^2}{a^2+b^2}\right)^2 + \left(b\left(1 - \frac{ab^2}{(a^2+b^2)a}\right)\right)^2$

$= \frac{a^2b^2}{a^2+b^2}$

Short Trick



minimum distance from origin to the line will be perpendicular distance

$\Rightarrow \left|\frac{1}{\sqrt{1/a^2 + 1/b^2}}\right| = \sqrt{x^2+y^2}$

$\Rightarrow x^2 + y^2 = \frac{a^2b^2}{a^2+b^2}$

FREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.com

Q.20 The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x- axis, is –

[AIEEE 2010]

(A) $y = 0$ (B) $y = 1$ (C) $y = 2$ (D) $y = 3$

Sol. **[D]**

Proper Method

$y = x + \frac{4}{x^2}$

$\frac{dy}{dx} = 1 - \frac{8}{x^3}$

$\because$ tangent is || to x axis

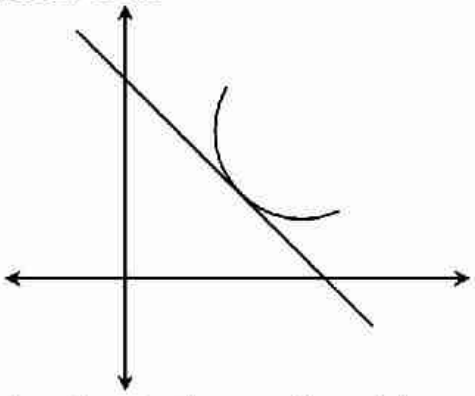
$\therefore \quad 1 - \frac{4}{x^3} = 0 \Rightarrow x = 2$

When $x = 2$

$y = 2 + \frac{4}{2^2} \Rightarrow y = 3$

$\therefore$ equation of tangent is $y = 3$

Short Trick



According to diagram slope of tangent is negative

$f'(x) = 2x + b$

$f'(1) = 2 + b$

by option $2 + b$ will be negative only when $b = -3$

FREE BOOKS FOR JEE & NEET
join telegram channel t.me/iitjeeadv
our website freebooksforjeeneet.blogspot.com